

Quark-antiquark composite systems: the Bethe–Salpeter equation in the spectral-integration technique in case of different quark masses

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Abstract

The Bethe–Salpeter equations for the quark-antiquark composite systems with different quark masses, such as $q\bar{s}$ (with $q = u, d$), $q\bar{Q}$ and $s\bar{Q}$ (with $Q = c, b$), are written in terms of spectral integrals. For mesons characterized by the mass M , spin J and radial quantum number n , the equations are written for the (n, M^2) -trajectories with fixed J . The mixing between states with different quark spin S and angular momentum L are also discussed.

1 Introduction

The relativistic description of quark–antiquark states is a necessary step for meson systematics and the search for exotic states. The standard way to take account of relativistic effects is to use the Bethe–Salpeter equation [1]. Different versions of the Bethe–Salpeter equations applied to the description of quark–antiquark systems may be found in [4, 5, 6, 7, 8]. In the present paper we develop the approach suggested in [2] for the Bethe–Salpeter equation written for quark–antiquark systems in terms of spectral

integrals. In [2], the systems of quarks with equal masses have been considered such as $u\bar{u}, u\bar{d}, d\bar{d}, s\bar{s}$. In this paper, the systems with unequal masses like $q\bar{s}$ ($q = u, d$) and $q\bar{Q}$ with $Q = c, b$ are treated.

Detailed presentation of the spectral integration method as well as the emphasis on its advantages was given in [2], so we need not to repeat ourselves. Let us only stress a particular feature of the method: it is rather easy to control the quark–gluonium content of the composite system that is rather important for the search for exotics. Another advantage consists in the easy treatment of the systems with high spins.

Similarly to [2], we present here the equations for the group of states laying on the (n, M^2) trajectory. Such trajectories are linear, they are suitable for the reconstruction of interaction between quarks at large distances. We hope, by investigating high-spin quark–antiquark states, to obtain decisive information on the structure of forces in the region of $r \sim R_{confinement}$.

In this present paper we present final formulae for the (n, M^2) -trajectories, the details of the calculations may be found in [2]. It should be immediately emphasized that the case of different masses requires more cumbersome calculations. In particular, the mixing of states with $(J = L; S = 0)$ and $(J = L; S = 1)$ should be accounted for (here L and S are the orbital moment and spin of quarks, respectively).

Note that for systems with the equal quark masses we have already obtained numerical results for a set of the (n, M^2) -trajectories such as: $a_0, a_1, a_2, \pi, \rho, b_1$. So we hope that we have elaborated rather efficient technique allowing us to find out realistic wave functions for the quark–antiquark systems. Certain aspects of numerical solutions of the Bethe–Salpeter equations in terms of the spectral integrals are discussed in [9].

The paper is organized as follows. In Section 2 we define the quantities entering the Bethe–Salpeter equation: for equal masses they were introduced in [2], now we expand the definition for unequal masses. In Section 3, the equations for the (n, M^2) -trajectories are considered for two different cases, for $(J = L; S = 0)$ and $(J = L; S = 1)$ states. The technicalities related to the trace calculations of loop diagrams as well as trace factor convolutions are considered in Appendices A and B.

2 Quark-antiquark composite systems

In the spectral integral technique, the Bethe–Salpeter equation for the wave function of the $q_1 \bar{q}_2$ system with the total momentum J , angular momentum $L = |\vec{J} - \vec{S}|$ and quark-antiquark spin S can be schematically written as:

$$\begin{aligned} & (s - M^2) \hat{\Psi}_{(n) \mu_1 \dots \mu_J}^{(S, L, J)}(k_\perp) = \\ & = \int \frac{d^3 k'_\perp}{(2\pi)^3} \Phi(s') \hat{V}(s, s', (k_\perp k'_\perp)) (\hat{k}'_1 + m_1) \hat{\Psi}_{(n) \mu_1 \dots \mu_J}^{(S, L, J)}(k'_\perp) (-\hat{k}'_2 + m_2), \end{aligned} \quad (1)$$

where the quarks are mass-on-shell: $k_1^2 = k_1'^2 = m_1^2$ and $k_2^2 = k_2'^2 = m_2^2$. We use the following notations:

$$k = \frac{1}{2}(k_1 - k_2), \quad P = k_1 + k_2, \quad k' = \frac{1}{2}(k'_1 - k'_2), \quad P' = k'_1 + k'_2, \quad (2)$$

$$P^2 = s, \quad P'^2 = s', \quad k_\mu^\perp = k_\nu g_{\nu\mu}^\perp, \quad k'_\mu{}^\perp = k'_\nu g_{\nu\mu}^\perp.$$

The phase space integral is written as

$$ds' \frac{d^3 k'_1}{2k'_{10}} \frac{d^3 k'_2}{2k'_{20}} \delta^{(4)}(P' - k'_1 - k'_2) = \frac{d^3 k'_\perp}{(2\pi)^3} \Phi(s'), \quad (3)$$

$$\Phi(s') = \frac{2s' \sqrt{s'}}{s'^2 - (m_1^2 - m_2^2)^2}.$$

The wave function reads:

$$\hat{\Psi}_{(n) \mu_1 \dots \mu_J}^{(S, L, J)}(k_\perp) = Q_{\mu_1 \dots \mu_J}^{(S, L, J)}(k_\perp) \psi_n^{(S, L, J)}(k_\perp^2). \quad (4)$$

Here $Q_{\mu_1 \dots \mu_J}^{(S, L, J)}(k_\perp)$ are the moment-operators for fermion-antifermion systems [3] defined as follows:

$$\begin{aligned} Q_{\mu_1 \mu_2 \dots \mu_J}^{(0, J, J)}(k_\perp) &= i\gamma_5 X_{\mu_1 \dots \mu_J}^{(J)}(k_\perp), \\ Q_{\mu_1 \dots \mu_J}^{(1, J+1, J)}(k_\perp) &= \gamma_\alpha^\perp X_{\mu_1 \dots \mu_J \alpha}^{(J+1)}(k_\perp), \\ Q_{\mu_1 \dots \mu_J}^{(1, J, J)}(k_\perp) &= \varepsilon_{\alpha \nu_1 \nu_2 \nu_3} \gamma_\alpha^\perp P_{\nu_1} Z_{\nu_2 \mu_1 \dots \mu_J, \nu_3}^{(J)}(k_\perp), \\ Q_{\mu_1 \dots \mu_J}^{(1, J-1, J)}(k_\perp) &= \gamma_\alpha^\perp Z_{\mu_1 \dots \mu_J, \alpha}^{(J-1)}(k_\perp), \end{aligned} \quad (5)$$

where

$$\begin{aligned}
X_{\mu_1 \dots \mu_J}^{(J)}(k_\perp) &= \frac{(2J-1)!!}{J!} \left[k_{\mu_1}^\perp k_{\mu_2}^\perp k_{\mu_3}^\perp k_{\mu_4}^\perp \dots k_{\mu_J}^\perp - \right. \\
&- \frac{k_\perp^2}{2J-1} \left(g_{\mu_1 \mu_2}^\perp k_{\mu_3}^\perp k_{\mu_4}^\perp \dots k_{\mu_J}^\perp + g_{\mu_1 \mu_3}^\perp k_{\mu_2}^\perp k_{\mu_4}^\perp \dots k_{\mu_J}^\perp + \dots \right) + \\
&+ \frac{k_\perp^4}{(2J-1)(2J-3)} \left(g_{\mu_1 \mu_2}^\perp g_{\mu_3 \mu_4}^\perp k_{\mu_5}^\perp k_{\mu_6}^\perp \dots k_{\mu_J}^\perp + \right. \\
&\left. + g_{\mu_1 \mu_2}^\perp g_{\mu_3 \mu_5}^\perp k_{\mu_4}^\perp k_{\mu_6}^\perp \dots k_{\mu_J}^\perp + \dots \right) + \dots \Big] , \\
Z_{\mu_1 \dots \mu_J, \alpha}^{(J-1)}(k_\perp) &= \frac{2J-1}{L^2} \left(\sum_{i=1}^J X_{\mu_1 \dots \mu_{i-1} \mu_{i+1} \dots \mu_J}^{(J-1)}(k_\perp) g_{\mu_i \alpha}^\perp - \right. \\
&\left. - \frac{2}{2J-1} \sum_{\substack{i,j=1 \\ i < j}}^J g_{\mu_i \mu_j}^\perp X_{\mu_1 \dots \mu_{i-1} \mu_{i+1} \dots \mu_{j-1} \mu_{j+1} \dots \mu_J}^{(J-1)}(k_\perp) \right) .
\end{aligned} \tag{6}$$

The potential operator can be represented as a sum of the t -channel operators:

$$\widehat{V}(s, s', (k_\perp k'_\perp)) = \sum_I V_I^{(0)}(s, s', (k_\perp k'_\perp)) \widehat{O}_I \otimes \widehat{O}_I , \tag{7}$$

$$\widehat{O}_I = \mathbf{I}, \quad \gamma_\mu, \quad i\sigma_{\mu\nu}, \quad i\gamma_\mu \gamma_5, \quad \gamma_5.$$

To write the spectral integral equations we are to transform the t -channel potential operator $\widehat{V}(s, s', (k_\perp k'_\perp))$ into the s -channel ones as follows:

$$\widehat{V}(s, s', (k_\perp k'_\perp)) = \sum_I \sum_c \widehat{V}_I^{(0)}(s, s', (k_\perp k'_\perp)) C_{Ic} (\widehat{F}_c \otimes \widehat{F}_c), \tag{8}$$

where C_{Ic} are coefficients of the Fierz matrix:

$$C = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{8} & \frac{1}{4} & \frac{1}{4} \\ 1 & -\frac{1}{2} & 0 & \frac{1}{2} & -1 \\ 3 & 0 & -\frac{1}{2} & 0 & 3 \\ 1 & \frac{1}{2} & 0 & -\frac{1}{2} & -1 \\ \frac{1}{4} & -\frac{1}{4} & \frac{1}{8} & -\frac{1}{4} & \frac{1}{4} \end{pmatrix}. \tag{9}$$

Here the summation is assumed in the $i\sigma_{\mu\nu} \otimes i\sigma_{\mu\nu}$ structure for all indices. Denoting

$$V_c(s, s', (k_\perp k'_\perp)) = \sum_I \hat{V}_I^{(0)}(s, s', (k_\perp k'_\perp)) C_{Ic}, \quad (10)$$

we have

$$\begin{aligned} \hat{V}(s, s', (k_\perp k'_\perp)) &= \sum_c (\hat{F}_c \otimes \hat{F}_c) V_c(s, s', (k_\perp k'_\perp)) = \\ &= (\mathbf{I} \otimes \mathbf{I}) V_S(s, s', (k_\perp k'_\perp)) + (\gamma_\mu \otimes \gamma_\mu) V_V(s, s', (k_\perp k'_\perp)) + (i\sigma_{\mu\nu} \otimes i\sigma_{\mu\nu}) \times \\ &\times V_T(s, s', (k_\perp k'_\perp)) + (i\gamma_\mu \gamma_\nu \otimes i\gamma_\mu \gamma_\nu) V_A(s, s', (k_\perp k'_\perp)) + (\gamma_5 \otimes \gamma_5) V_P(s, s', (k_\perp k'_\perp)). \end{aligned} \quad (11)$$

Let us multiply Eq. (1) by the operator $Q_{\mu_1 \dots \mu_J}^{(S, L, J)}(k_\perp)$ and convolute over the spin-momentum indices:

$$\begin{aligned} &(s - M^2) Sp \left[\hat{\Psi}_{(n) \mu_1 \dots \mu_J}^{(S, L, J)}(k_\perp) (\hat{k}_1 + m_1) Q_{\mu_1 \dots \mu_J}^{(S, L, J)}(k_\perp) (-\hat{k}_2 + m_2) \right] \\ &= \sum_c Sp \left[\hat{F}_c (\hat{k}_1 + m_1) Q_{\mu_1 \dots \mu_J}^{(S, L, J)}(k_\perp) (-\hat{k}_2 + m_2) \right] \int \frac{d^3 k'_\perp}{(2\pi)^3} \Phi(s') \times \\ &\times V_c(s, s', (k_\perp k'_\perp)) Sp \left[(\hat{k}'_1 + m_1) \hat{F}_c (-\hat{k}'_2 + m_2) \hat{\Psi}_{(n) \mu_1 \dots \mu_J}^{(S, L, J)}(k'_\perp) \right]. \end{aligned} \quad (12)$$

We have four states with the $q_1 \bar{q}_2$ spins $S = 0$ and $S = 1$:

- 1) $S = 0$; $L = J$,
- 2) $S = 1$; $L = J + 1, J, J - 1$,

which are mixed and form two final states. The wave functions read:

for $S = 0, 1, J = L$,

$$\hat{\Psi}_{(n) \mu_1 \dots \mu_J}^{(S_i, J, J)}(k_\perp) = C_i \hat{\Psi}_{(n) \mu_1 \dots \mu_J}^{(0, J, J)}(k_\perp) + D_i \hat{\Psi}_{(n) \mu_1 \dots \mu_J}^{(1, J, J)}(k_\perp) \quad (13)$$

where C_i and D_i are the mixing coefficients with $i = 1, 2$.

for $S = 1, L = J \pm 1, J$,

$$\hat{\Psi}_{(n) \mu_1 \dots \mu_J}^{(1, (J \pm 1)_j, J)}(k_\perp) = A_j \hat{\Psi}_{(n) \mu_1 \dots \mu_J}^{(1, J-1, J)}(k_\perp) + B_j \hat{\Psi}_{(n) \mu_1 \dots \mu_J}^{(1, J+1, J)}(k_\perp). \quad (14)$$

where A_j and B_j are the mixing coefficients with $j = 1, 2$.

These wave functions are normalized:

$$\begin{aligned} &\int \frac{d^3 k_\perp}{(2\pi)^3} \Phi(s) (-1) Sp \left[\hat{\Psi}_{(n') \mu_1 \dots \mu_J}^{(S', L'_j, J')}(k_\perp) (\hat{k}_1 + m_1) \hat{\Psi}_{(n) \mu_1 \dots \mu_J}^{(S, L_j, J)}(k_\perp) (-\hat{k}_2 + m_2) \right] \\ &= (-1)^J \delta_{S'S} \delta_{L'_j L_j} \delta_{J'J} \delta_{n'n}. \end{aligned} \quad (15)$$

3 Equations for (n, M^2) trajectories

In this section we write the trajectories for $(J = L, S = 0, 1)$ and $(J = L \pm 1, S = 1)$ states.

3.1 The equation for the $(S = 0, 1, J = L)$ -state

There are two equations for the two states with $S = 0, 1$ and $J = L$. Their wave functions are denoted as $C_i \hat{\Psi}_{(n)\mu_1 \dots \mu_J}^{(0,J,J)}(k_\perp) + D_i \hat{\Psi}_{(n)\mu_1 \dots \mu_J}^{(1,J,J)}(k_\perp)$, with $i = 1, 2$. These wave functions are orthogonal to each other. Normalization and orthogonality conditions give three constraints for four mixing parameters C_i and D_i .

Each wave function obeys two equations:

$$\begin{aligned}
 & (s - M^2) X_{\mu_1 \dots \mu_J}^{(J)}(k_\perp) Sp \left[i\gamma_5 \gamma_\alpha^\perp (\hat{k}_1 + m_1) i\gamma_5 (-\hat{k}_2 + m_2) \right] \times \quad (16) \\
 & \times \left(C_i X_{\mu_1 \dots \mu_J}^{(J)}(k_\perp) \psi_n^{(0,J,J)}(k_\perp^2) + D_i \varepsilon_{\alpha\nu_1\nu_2\nu_3} P_{\nu_1} Z_{\nu_2\mu_1 \dots \mu_J, \nu_3}^{(J)}(k_\perp) \psi_n^{(1,J,J)}(k_\perp^2) \right) = \\
 & = X_{\mu_1 \dots \mu_J}^{(J)}(k_\perp) \sum_c Sp \left[\hat{F}_c (\hat{k}_1 + m_1) i\gamma_5 (-\hat{k}_2 + m_2) \right] \times \\
 & \times \int \frac{d^3 k'_\perp}{(2\pi)^3} \Phi(s') V_c(s, s', (k_\perp k'_\perp)) Sp \left[i\gamma_5 \gamma_{\alpha'}^\perp (\hat{k}'_1 + m_1) \hat{F}_c (-\hat{k}'_2 + m_2) \right] \times \\
 & \times \left(C_i X_{\mu_1 \dots \mu_J}^{(J)}(k'_\perp) \psi_n^{(0,J,J)}(k'^2_\perp) + D_i \varepsilon_{\alpha'\nu_1\nu_2\nu_3} P'_{\nu_1} Z_{\nu_2\mu_1 \dots \mu_J, \nu_3}^{(J)}(k'_\perp) \psi_n^{(1,J,J)}(k'^2_\perp) \right)
 \end{aligned}$$

and

$$\begin{aligned}
 & (s - M^2) \varepsilon_{\beta\nu_1\nu_2\nu_3} P_{\nu_1} Z_{\nu_2\mu_1 \dots \mu_J, \nu_3}^{(J)}(k_\perp) Sp \left[i\gamma_5 \gamma_\alpha^\perp (\hat{k}_1 + m_1) \gamma_\beta^\perp (-\hat{k}_2 + m_2) \right] \times (17) \\
 & \times \left(C_i X_{\mu_1 \dots \mu_J}^{(J)}(k_\perp) \psi_n^{(0,J,J)}(k_\perp^2) + D_i \varepsilon_{\alpha\nu_1\nu_2\nu_3} P_{\nu_1} Z_{\nu_2\mu_1 \dots \mu_J, \nu_3}^{(J)}(k_\perp) \psi_n^{(1,J,J)}(k_\perp^2) \right) = \\
 & = \varepsilon_{\beta'\nu_1\nu_2\nu_3} P_{\nu_1} Z_{\nu_2\mu_1 \dots \mu_J, \nu_3}^{(J)}(k_\perp) \sum_c Sp \left[\hat{F}_c (\hat{k}_1 + m_1) \gamma_{\beta'}^\perp (-\hat{k}_2 + m_2) \right] \times \\
 & \times \int \frac{d^3 k'_\perp}{(2\pi)^3} \Phi(s') V_c(s, s', (k_\perp k'_\perp)) Sp \left[i\gamma_5 \gamma_{\alpha'}^\perp (\hat{k}'_1 + m_1) \hat{F}_c (-\hat{k}'_2 + m_2) \right] \times \\
 & \times \left(C_i X_{\mu_1 \dots \mu_J}^{(J)}(k'_\perp) \psi_n^{(0,J,J)}(k'^2_\perp) + D_i \varepsilon_{\alpha'\nu_1\nu_2\nu_3} P'_{\nu_1} Z_{\nu_2\mu_1 \dots \mu_J, \nu_3}^{(J)}(k'_\perp) \psi_n^{(1,J,J)}(k'^2_\perp) \right)
 \end{aligned}$$

Now consider the left-hand side of the equation. Using the traces written in Appendix A and convolution of operators from Appendix B, we have:

$$X_{\mu_1 \dots \mu_J}^{(J)}(k_\perp) Sp \left[i\gamma_5(\widehat{k}_1 + m_1) i\gamma_5(-\widehat{k}_2 + m_2) \right] X_{\mu_1 \dots \mu_J}^{(J)}(k_\perp) = -2(s - \Delta^2) \alpha(J) k_\perp^{2J},$$

$$X_{\mu_1 \dots \mu_J}^{(J)}(k_\perp) Sp \left[\gamma_\alpha^\perp(\widehat{k}_1 + m_1) i\gamma_5(-\widehat{k}_2 + m_2) \right] \varepsilon_{\alpha\nu_1\nu_2\nu_3} P_{\nu_1} Z_{\nu_2\mu_1 \dots \mu_J, \nu_3}^{(J)}(k_\perp) = 0 \quad (18)$$

Here and below we use the following notations: $\delta = m_2 - m_1$, $\sigma = m_2 + m_1$. Also the left-hand side of (17) contains two convolutions:

$$\varepsilon_{\beta\nu_1\nu_2\nu_3} P_{\nu_1} Z_{\nu_2\mu_1 \dots \mu_J, \nu_3}^{(J)}(k_\perp) Sp \left[i\gamma_5(\widehat{k}_1 + m_1) \gamma_\beta^\perp(-\widehat{k}_2 + m_2) \right] X_{\mu_1 \dots \mu_J}^{(J)}(k_\perp) = 0, \quad (19)$$

$$\begin{aligned} & \varepsilon_{\beta\nu_1\nu_2\nu_3} P_{\nu_1} Z_{\nu_2\mu_1 \dots \mu_J, \nu_3}^{(J)}(k_\perp) Sp \left[\gamma_\alpha^\perp(\widehat{k}_1 + m_1) \gamma_\beta^\perp(-\widehat{k}_2 + m_2) \right] \times \\ & \times \varepsilon_{\alpha\nu_1\nu_2\nu_3} P_{\nu_1} Z_{\nu_2\mu_1 \dots \mu_J, \nu_3}^{(J)}(k_\perp) = -2s(s - \Delta^2) \frac{J(2J+3)^2}{(J+1)^3} \alpha(J) k_\perp^{2J}. \end{aligned}$$

The right-hand side of the equation is calculated in two steps. First, we summarize over c :

$$\begin{aligned} A(s, s', (k_\perp k'_\perp)) &= \sum_{c=T, A, P} A_c(s, s', (k_\perp k'_\perp)) V_c(s, s', (k_\perp k'_\perp)) = \quad (20) \\ &= \sum_{c=T, A, P} Sp \left[\widehat{F}_c(\widehat{k}_1 + m_1) i\gamma_5(-\widehat{k}_2 + m_2) \right] Sp \left[i\gamma_5(\widehat{k}'_1 + m_1) \widehat{F}_c(-\widehat{k}'_2 + m_2) \right] \times \\ & \quad \times V_c(s, s', (k_\perp k'_\perp)), \end{aligned}$$

$$\begin{aligned} B_{\beta'\alpha'}(s, s', (k_\perp k'_\perp)) &= \sum_{c=T, A, V, S} (B_c)_{\beta'\alpha'}(s, s', (k_\perp k'_\perp)) V_c(s, s', (k_\perp k'_\perp)) = \quad (21) \\ &= \sum_{c=T, A, V, S} Sp \left[\widehat{F}_c(\widehat{k}_1 + m_1) \gamma_{\beta'}^\perp(-\widehat{k}_2 + m_2) \right] Sp \left[\gamma_{\alpha'}^\perp(\widehat{k}'_1 + m_1) \widehat{F}_c(-\widehat{k}'_2 + m_2) \right] \times \\ & \quad \times V_c(s, s', (k_\perp k'_\perp)), \end{aligned}$$

and

$$C_{\alpha'}(s, s', (k_\perp k'_\perp)) = \sum_{c=T, A} C_{\alpha'}^c(s, s', (k_\perp k'_\perp)) V_c(s, s', (k_\perp k'_\perp)) = \quad (22)$$

$$= \sum_{c=T,A} Sp \left[\hat{F}_c (\hat{k}_1 + m_1) i \gamma_5 (-\hat{k}_2 + m_2) \right] Sp \left[\gamma_{\alpha'}^\perp (\hat{k}'_1 + m_1) \hat{F}_c (-\hat{k}'_2 + m_2) \right] \times \\ \times V_c (s, s', (k_\perp k'_\perp)) .$$

In Appendix A the trace calculations are presented, and the values $A_c (s, s', (k_\perp k'_\perp))$, $C_c (s, s', (k_\perp k'_\perp))$ are given. So after the summation, A_c , B_c , C_c are written as follows :

$$A (s, s', (k_\perp k'_\perp)) = \sum_{c=T,A,P} A_c (s, s', (k_\perp k'_\perp)) V_c (s, s', (k_\perp k'_\perp)) = \quad (23)$$

$$= -16(k_\perp k'_\perp) \left[2\sqrt{ss'} V_T (s, s', (k_\perp k'_\perp)) + \Delta^2 V_A (s, s', (k_\perp k'_\perp)) \right] - \\ -4(s - \Delta^2)(s' - \Delta^2) \left[\frac{\sigma\sigma'}{\sqrt{ss'}} V_A (s, s', (k_\perp k'_\perp)) + V_P (s, s', (k_\perp k'_\perp)) \right] ,$$

$$B_{\beta'\alpha'} (s, s', (k_\perp k'_\perp)) = \sum_{c=T,A,V,S} (B_c)_{\beta'\alpha'} (s, s', (k_\perp k'_\perp)) V_c (s, s', (k_\perp k'_\perp)) = \quad (24)$$

$$= 4 g_{\beta'\alpha'}^\perp \left[(s - \Delta^2)(s' - \Delta^2) \left(V_V (s, s', (k_\perp k'_\perp)) + 2 \frac{\sigma^2}{\sqrt{ss'}} V_T (s, s', (k_\perp k'_\perp)) \right) + \right. \\ \left. + 4\sqrt{k_\perp^2} \sqrt{k'^2_\perp} z \left(\sqrt{ss'} V_A (s, s', (k_\perp k'_\perp)) + 2\Delta^2 V_T (s, s', (k_\perp k'_\perp)) \right) \right] + \\ + 16 k_{\beta'}^\perp k_{\alpha'}'^\perp \left[\left(\sigma^2 V_S (s, s', (k_\perp k'_\perp)) + \frac{\sigma^2 \Delta^2}{\sqrt{ss'}} V_V (s, s', (k_\perp k'_\perp)) \right) + \right. \\ \left. + 4\sqrt{k_\perp^2} \sqrt{k'^2_\perp} z V_V (s, s', (k_\perp k'_\perp)) \right] - \\ - 16 k_{\beta'}'^\perp k_{\alpha'}^\perp \left[\sqrt{ss'} V_A (s, s', (k_\perp k'_\perp)) + 2\Delta^2 V_T (s, s', (k_\perp k'_\perp)) \right] + \\ + 16 k_{\beta'}^\perp k_{\alpha'}^\perp (s' - \Delta^2) V_V (s, s', (k_\perp k'_\perp)) + 16 k_{\beta'}'^\perp k_{\alpha'}'^\perp (s - \Delta^2) V_V (s, s', (k_\perp k'_\perp)) ,$$

and

$$C_{\alpha'} (s, s', (k_\perp k'_\perp)) = \sum_{c=T,A} C_{\alpha'}^c (s, s', (k_\perp k'_\perp)) V_c (s, s', (k_\perp k'_\perp)) = \quad (25)$$

$$= 8 [2\Delta \varepsilon_{\alpha' k k' P'} V_A (s, s', (k_\perp k'_\perp)) + \sigma \varepsilon_{\alpha' P k' P'} V_A (s, s', (k_\perp k'_\perp)) + \\ + 4\Delta \varepsilon_{\alpha' P k k'} V_T (s, s', (k_\perp k'_\perp)) + 2\sigma \varepsilon_{\alpha' P k P'} V_T (s, s', (k_\perp k'_\perp))]$$

Here we used a short notation: $\varepsilon_{\alpha'kk'P'} \equiv k_\beta k'_\mu P'_\nu \varepsilon_{\alpha'\beta\mu\nu}$. Second, the convolution of operators is performed by using equations of Appendix B and recurrent formulae for the Legendre polynomials:

$$zP_J(z) = \frac{J+1}{2J+1}P_{J+1}(z) + \frac{J}{2J+1}P_{J-1}(z),$$

that allows us to represent the Bethe–Salpeter equation in terms of the Legendre polynomials. As a result we get:

$$\begin{aligned} & X_{\mu_1 \dots \mu_J}^{(J)}(k_\perp) A(s, s', (k_\perp k'_\perp)) X_{\mu_1 \dots \mu_J}^{(J)}(k'_\perp) = -4\alpha(J) \left(\sqrt{k_\perp^2} \sqrt{k'^2_\perp} \right)^J \times \quad (26) \\ & \times \left[4 \frac{J+1}{2J+1} \sqrt{k_\perp^2} \sqrt{k'^2_\perp} \left(2\sqrt{ss'} V_T(s, s', (k_\perp k'_\perp)) + \Delta^2 V_A(s, s', (k_\perp k'_\perp)) \right) P_{J+1}(z) + \right. \\ & + (s - \Delta^2)(s' - \Delta^2) \left(\frac{\sigma^2}{\sqrt{ss'}} V_A(s, s', (k_\perp k'_\perp)) + V_P(s, s', (k_\perp k'_\perp)) \right) P_J(z) + \\ & \left. + 4 \frac{J}{2J+1} \sqrt{k_\perp^2} \sqrt{k'^2_\perp} \left(2\sqrt{ss'} V_T(s, s', (k_\perp k'_\perp)) + \Delta^2 V_A(s, s', (k_\perp k'_\perp)) \right) P_{J-1}(z) \right], \end{aligned}$$

and

$$\begin{aligned} & X_{\mu_1 \dots \mu_J}^{(J)}(k_\perp) C_{\alpha'}(s, s', (k_\perp k'_\perp)) \varepsilon_{\alpha' \nu_1 \nu_2 \nu_3} P'_{\nu_1} Z_{\nu_2 \mu_1 \dots \mu_J, \nu_3}^{(J)}(k'_\perp) = \quad (27) \\ & = 16 \frac{2J+3}{J+1} \alpha(J) \left(\sqrt{k_\perp^2} \sqrt{k'^2_\perp} \right)^{J+1} \times \\ & \times \left(\frac{2}{J+1} P'_J(z) - J P_{J+1}(z) \right) \left(s' \Delta V_A(s, s', (k_\perp k'_\perp)) + 2\Delta \sqrt{ss'} V_T(s, s', (k_\perp k'_\perp)) \right). \end{aligned}$$

For the right-hand side of (17) we have:

$$\begin{aligned} & \varepsilon_{\beta' \nu_1 \nu_2 \nu_3} P_{\nu_1} Z_{\nu_2 \mu_1 \dots \mu_J, \nu_3}^{(J)}(k_\perp) C_{\beta'}(s, s', (k_\perp k'_\perp)) X_{\mu_1 \dots \mu_J}^{(J)}(k'_\perp) = \quad (28) \\ & = 16 \frac{2J+3}{J+1} \alpha(J) \left(\sqrt{k_\perp^2} \sqrt{k'^2_\perp} \right)^{J+1} \times \\ & \times \left(\frac{2}{J+1} P'_J(z) - J P_{J+1}(z) \right) \left(s \Delta V_A(s, s', (k_\perp k'_\perp)) + 2\Delta \sqrt{ss'} V_T(s, s', (k_\perp k'_\perp)) \right) \end{aligned}$$

and

$$\varepsilon_{\beta' \nu_1 \nu_2 \nu_3} P_{\nu_1} Z_{\nu_2 \mu_1 \dots \mu_J, \nu_3}^{(J)}(k_\perp) B_{\beta' \alpha'}(s, s', (k_\perp k'_\perp)) \varepsilon_{\alpha' \nu_1 \nu_2 \nu_3} P'_{\nu_1} Z_{\nu_2 \mu_1 \dots \mu_J, \nu_3}^{(J)}(k'_\perp) = \quad (29)$$

$$\begin{aligned}
&= -4\sqrt{ss'} \frac{J(2J+3)^2}{(J+1)^3} \alpha(J) \left(\sqrt{k_\perp^2} \sqrt{k_\perp'^2} \right)^J \times \\
&\times \left[4 \frac{J}{2J+1} \sqrt{k_\perp^2} \sqrt{k_\perp'^2} \left(\sqrt{ss'} V_A(s, s', (k_\perp k'_\perp)) + 2\Delta^2 V_T(s, s', (k_\perp k'_\perp)) \right) P_{J+1}(z) \right. \\
&+ (s - \Delta^2)(s' - \Delta^2) \left(V_V(s, s', (k_\perp k'_\perp)) + 2 \frac{\sigma^2}{\sqrt{ss'}} V_T(s, s', (k_\perp k'_\perp)) \right) P_J(z) + \\
&\left. + 4 \frac{J+1}{2J+1} \sqrt{k_\perp^2} \sqrt{k_\perp'^2} \left(\sqrt{ss'} V_A(s, s', (k_\perp k'_\perp)) + 2\Delta^2 V_T(s, s', (k_\perp k'_\perp)) \right) P_{J-1}(z) \right].
\end{aligned}$$

Expanding the interaction block in the Legendre polynomials series,

$$V_c(s, s', (k_\perp k'_\perp)) = \sum_J V_c^{(J)}(s, s') P_J(z) = \quad (30)$$

$$= \sum_J \tilde{V}_c^{(J)}(s, s') \alpha(J) \left(-\sqrt{k_\perp^2} \sqrt{k_\perp'^2} \right)^J P_J(z),$$

and integrating over angle variables in the right-hand side by taking account of the standard normalization condition $\int_{-1}^1 dz/2 P_J^2(z) = 1/(2J+1)$, we have finally:

$$\begin{aligned}
&(s - M^2) \left[(s - \Delta^2) \psi_n^{(0,J,J)}(s) C_i \right] = \quad (31) \\
&= \int_{(m_1+m_2)^2}^{\infty} \frac{ds'}{\pi} \rho(s') 2 (-k_\perp'^2)^J \psi_n^{(0,J,J)}(s') C_j \times \\
&\times \left[-4 \xi(J+1) \frac{J+1}{2J+1} k_\perp^2 k_\perp'^2 \left(2\sqrt{ss'} \tilde{V}_T^{(J+1)}(s, s') + \Delta^2 \tilde{V}_A^{(J+1)}(s, s') \right) + \right. \\
&\quad + \xi(J) (s - \Delta^2)(s' - \Delta^2) \left(\frac{\sigma^2}{\sqrt{ss'}} \tilde{V}_A^{(J)}(s, s') + \tilde{V}_P^{(J)}(s, s') \right) - \\
&\quad \left. - 4 \xi(J-1) \frac{J}{2J+1} \left(2\sqrt{ss'} \tilde{V}_T^{(J-1)}(s, s') + \Delta^2 \tilde{V}_A^{(J-1)}(s, s') \right) \right] - \\
&\quad - \int_{(m_1+m_2)^2}^{\infty} \frac{ds'}{\pi} \rho(s') 8 \frac{2J+3}{J+1} k_\perp^2 (-k_\perp'^2)^{J+1} \psi_n^{(1,J,J)}(s') D_j \times \\
&\times \left[\frac{2}{J+1} \sum_a (2J - 4a - 1) \xi(J - 2a - 1) (-\sqrt{k_\perp^2} \sqrt{k_\perp'^2})^{-2(a+1)} \times \right.
\end{aligned}$$

$$\begin{aligned} & \times \left(s' \Delta \tilde{V}_A^{(J-2a-1)}(s, s') + 2\Delta \sqrt{ss'} \tilde{V}_T^{(J-2a-1)}(s, s') \right) - \\ & - J \xi(J+1) \left(s' \Delta \tilde{V}_A^{(J+1)}(s, s') + 2\Delta' \sqrt{ss'} \tilde{V}_T^{(J+1)}(s, s') \right), \end{aligned}$$

where

$$P'_J(z) = \sum_a (2J - 4a - 1) P_{J-2a-1}(z). \quad (32)$$

The second equation, (17), reads:

$$\begin{aligned} & (s - M^2) \left[s(s - \Delta^2) \frac{J(2J+3)^2}{(J+1)^3} \psi_n^{(1,J,J)}(s) D_i \right] = \quad (33) \\ & = - \int_{(m_1+m_2)^2}^{\infty} \frac{ds'}{\pi} \rho(s') 8 \frac{2J+3}{J+1} k_{\perp}^2 (-k_{\perp}'^2)^{J+1} \psi_n^{(0,J,J)}(s') D_j \times \\ & \times \left(\frac{2}{J+1} \sum_a (2J - 4a - 1) \xi(J - 2a - 1) (-\sqrt{k_{\perp}^2} \sqrt{k_{\perp}'^2})^{-2(k+1)} \times \right. \\ & \times \left(s \Delta \tilde{V}_A^{(J-2a-1)}(s, s') + 2\Delta \sqrt{ss'} \tilde{V}_T^{(J-2a-1)}(s, s') \right) - \\ & - J \xi(J+1) \left(s \Delta \tilde{V}_A^{(J+1)}(s, s') + 2\Delta \sqrt{ss'} \tilde{V}_T^{(J+1)}(s, s') \right) + \\ & + \int_{(m_1+m_2)^2}^{\infty} \frac{ds'}{\pi} \rho(s') 2 \sqrt{ss'} \frac{J(2J+3)^2}{(J+1)^3} (-k_{\perp}'^2)^J \psi_n^{(1,J,J)}(s') D_j \times \\ & \times \left[-4 \xi(J+1) \frac{J}{2J+1} k_{\perp}^2 k_{\perp}'^2 \left(\sqrt{ss'} \tilde{V}_A^{(J+1)}(s, s') + 2\Delta^2 \tilde{V}_T^{(J+1)}(s, s') \right) + \right. \\ & + \xi(J) (s - \Delta^2)(s' - \Delta^2) \left(2 \frac{\sigma^2}{\sqrt{ss'}} \tilde{V}_T^{(J)}(s, s') + \tilde{V}_V^{(J)}(s, s') \right) - \\ & \left. - 4 \xi(J-1) \frac{J+1}{2J+1} \left(\sqrt{ss'} \tilde{V}_A^{(J-1)}(s, s') + 2\Delta^2 \tilde{V}_T^{(J-1)}(s, s') \right) \right]. \end{aligned}$$

The normalization and orthogonality conditions look as follows:

$$\int_{(m_2+m_1)^2}^{\infty} \frac{ds}{\pi} \rho(s) \left[C_i^2 \left(\psi_n^{(0,J,J)}(k_{\perp}^2) \right)^2 2\alpha(J) (-k_{\perp}^2)^J (s - \Delta^2) + \right. \quad (34)$$

$$+D_i^2 \left(\psi_n^{(1,J,J)}(k_\perp^2) \right)^2 2\alpha(J)(-k_\perp^2)^J s (s - \Delta^2) \frac{J(2J+3)^2}{(J+1)^3} \Big] = 1, \quad i = 1, 2,$$

and

$$\int_{(m_2+m_1)^2}^{\infty} \frac{ds}{\pi} \rho(s) \left[C_1 C_2 \left(\psi_n^{(0,J,J)}(k_\perp^2) \right)^2 2\alpha(J)(-k_\perp^2)^J (s - \Delta^2) + \right. \quad (35)$$

$$\left. + D_1 D_2 \left(\psi_n^{(1,J,J)}(k_\perp^2) \right)^2 2\alpha(J)(-k_\perp^2)^J s (s - \Delta^2) \frac{J(2J+3)^2}{(J+1)^3} \right] = 0.$$

3.2 The equations for the $(S = 1, J = L \pm 1)$ -states

We have two equations for the two states with $S = 1$ and $J = L \pm 1$. Their wave functions are denoted as $A_j \hat{\Psi}_{(n)\mu_1 \dots \mu_J}^{(1,J-1,J)}(k_\perp) + B_j \hat{\Psi}_{(n)\mu_1 \dots \mu_J}^{(1,J+1,J)}(k_\perp)$, with $j = 1, 2$. These wave functions are orthogonal. Normalization and orthogonality conditions give three constraints for four mixing parameters A_j and B_j .

Each wave function obeys two equations:

$$\begin{aligned} & (s - M^2) X_{\mu_1 \dots \mu_J \beta}^{(J+1)}(k_\perp) Sp \left[\gamma_\alpha^\perp(\hat{k}_1 + m_1) \gamma_\beta^\perp(-\hat{k}_2 + m_2) \right] \times \quad (36) \\ & \times \left(A_j Z_{\mu_1 \dots \mu_J, \alpha}^{(J-1)}(k_\perp) \psi_n^{(1,J-1,J)}(k_\perp^2) + B_j X_{\mu_1 \dots \mu_J \alpha}^{(J+1)}(k_\perp) \psi_n^{(1,J+1,J)}(k_\perp^2) \right) = \\ & = X_{\mu_1 \dots \mu_J \beta'}^{(J+1)}(k_\perp) \sum_c Sp \left[\hat{F}_c(\hat{k}_1 + m_1) \gamma_{\beta'}^\perp(-\hat{k}_2 + m_2) \right] \times \\ & \times \int \frac{d^3 k'_\perp}{(2\pi)^3} V_c(s, s', (k_\perp k'_\perp)) Sp \left[\gamma_{\alpha'}^\perp(\hat{k}'_1 + m_1) \hat{F}_c(-\hat{k}'_2 + m_2) \right] \times \\ & \times \left(A_j Z_{\mu_1 \dots \mu_J, \alpha'}^{(J-1)}(k'_\perp) \psi_n^{(1,J-1,J)}(k'^2_\perp) + B_j X_{\mu_1 \dots \mu_J \alpha'}^{(J+1)}(k'_\perp) \psi_n^{(1,J+1,J)}(k'^2_\perp) \right), \end{aligned}$$

and

$$\begin{aligned} & (s - M^2) Z_{\mu_1 \dots \mu_J, \beta}^{(J-1)}(k_\perp) Sp \left[\gamma_\alpha^\perp(\hat{k}_1 + m_1) \gamma_\beta^\perp(-\hat{k}_2 + m_2) \right] \times \quad (37) \\ & \times \left(A_j Z_{\mu_1 \dots \mu_J, \alpha}^{(J-1)}(k_\perp) \psi_n^{(1,J-1,J)}(k_\perp^2) + B_j X_{\mu_1 \dots \mu_J \alpha}^{(J+1)}(k_\perp) \psi_n^{(1,J+1,J)}(k_\perp^2) \right) = \\ & = Z_{\mu_1 \dots \mu_J, \beta'}^{(J-1)}(k_\perp) \sum_c Sp \left[\hat{F}_c(\hat{k}_1 + m_1) \gamma_{\beta'}^\perp(-\hat{k}_2 + m_2) \right] \times \end{aligned}$$

$$\begin{aligned} & \times \int \frac{d^3 k'_\perp}{(2\pi)^3} V_c(s, s', (k_\perp k'_\perp)) Sp \left[\gamma_\alpha^\perp(\hat{k}'_1 + m_1) \hat{F}_c(-\hat{k}'_2 + m_2) \right] \times \\ & \times \left(A_j Z_{\mu_1 \dots \mu_J, \alpha'}^{(J-1)}(k'_\perp) \psi_n^{(1, J-1, J)}(k'^2_\perp) + B_j X_{\mu_1 \dots \mu_J, \alpha'}^{(J+1)}(k'_\perp) \psi_n^{(1, J+1, J)}(k'^2_\perp) \right). \end{aligned}$$

Firstly, consider (36). In the left-hand side of (36) one has two convolutions:

$$\begin{aligned} & X_{\mu_1 \dots \mu_J, \beta}^{(J+1)}(k_\perp) Sp \left[\gamma_\alpha^\perp(\hat{k}_1 + m_1) \gamma_\beta^\perp(-\hat{k}_2 + m_2) \right] X_{\mu_1 \dots \mu_J, \alpha}^{(J+1)}(k_\perp) = \quad (38) \\ & = 2\alpha(J) k_\perp^{2(J+1)} \left[\frac{2J+1}{J+1} (s - \Delta^2) + 4k_\perp^2 \right], \end{aligned}$$

$$X_{\mu_1 \dots \mu_J, \beta}^{(J+1)}(k_\perp) Sp \left[\gamma_\alpha^\perp(\hat{k}_1 + m_1) \gamma_\beta^\perp(-\hat{k}_2 + m_2) \right] Z_{\mu_1 \dots \mu_J, \alpha}^{(J-1)}(k_\perp) = 8\alpha(J) k_\perp^{2(J+1)}.$$

Also the left-hand side of (37) contains two convolutions:

$$Z_{\mu_1 \dots \mu_J, \beta}^{(J-1)}(k_\perp) Sp \left[\gamma_\alpha^\perp(\hat{k}_1 + m_1) \gamma_\beta^\perp(-\hat{k}_2 + m_2) \right] X_{\mu_1 \dots \mu_J, \alpha}^{(J+1)}(k_\perp) = 8\alpha(J) k_\perp^{2(J+1)}, \quad (39)$$

$$\begin{aligned} & Z_{\mu_1 \dots \mu_J, \beta}^{(J-1)}(k_\perp) Sp \left[\gamma_\alpha^\perp(\hat{k}_1 + m_1) \gamma_\beta^\perp(-\hat{k}_2 + m_2) \right] Z_{\mu_1 \dots \mu_J, \alpha}^{(J-1)}(k_\perp) = \\ & = 2\alpha(J) k_\perp^{2(J-1)} \left[\frac{2J+1}{J} (s - \Delta^2) + 4k_\perp^2 \right]. \end{aligned}$$

The right-hand side of Eqs. (36) and (37) is determined by the convolutions of the trace factor $B_{\beta'\alpha'}(s, s', (k_\perp k'_\perp))$, see Eqs. (24), with angular-momentum wave functions; corresponding formulae may be found in Appendix B. Taking them into account one has for the right-hand side of (36):

$$\begin{aligned} & X_{\mu_1 \dots \mu_J, \beta'}^{(J+1)}(k_\perp) B_{\beta'\alpha'}(s, s', (k_\perp k'_\perp)) X_{\mu_1 \dots \mu_J, \alpha'}^{(J+1)}(k'_\perp) = 4\alpha(J) \left(\sqrt{k_\perp^2} \sqrt{k'^2_\perp} \right)^{J+1} \times (40) \\ & \times \left(\left[\frac{2J+1}{J+1} (s - \Delta^2) (s' - \Delta^2) \left(V_V(s, s', (k_\perp k'_\perp)) + 2 \frac{\sigma^2}{\sqrt{ss'}} V_T(s, s', (k_\perp k'_\perp)) \right) + \right. \right. \\ & + 4(s' - \Delta^2) k_\perp^2 V_V(s, s', (k_\perp k'_\perp)) + 4(s - \Delta^2) k'^2_\perp V_V(s, s', (k_\perp k'_\perp)) + \\ & \left. + 16 \frac{J+1}{2J+1} k_\perp^2 k'^2_\perp V_V(s, s', (k_\perp k'_\perp)) \right] P_{J+1}(z) + \\ & + 4 \left[\sigma^2 V_S(s, s', (k_\perp k'_\perp)) + \frac{\sigma^2 \Delta^2}{\sqrt{ss'}} V_V(s, s', (k_\perp k'_\perp)) + \right. \\ & \left. + \frac{J}{J+1} \left(\sqrt{ss'} V_A(s, s', (k_\perp k'_\perp)) + 2\Delta^2 V_T(s, s', (k_\perp k'_\perp)) \right) \right] \sqrt{k_\perp^2} \sqrt{k'^2_\perp} P_J(z) + \end{aligned}$$

$$+16\frac{J}{2J+1}k_{\perp}^2k'^2V_V(s,s',(k_{\perp}k'_{\perp}))P_{J-1}(z)),$$

and

$$\begin{aligned} X_{\mu_1\dots\mu_J,\beta'}^{(J+1)}(k_{\perp})B_{\beta'\alpha'}(s,s',(k_{\perp}k'_{\perp}))Z_{\mu_1\dots\mu_J,\alpha'}^{(J-1)}(k'_{\perp}) &= 16\alpha(J)k_{\perp}^2\left(\sqrt{k_{\perp}^2}\sqrt{k'_{\perp}^2}\right)^{J-1}\times(41) \\ &\times\left(\left[s-\Delta^2+4\frac{J+1}{2J+1}k_{\perp}^2\right]k'_{\perp}{}^2V_V(s,s',(k_{\perp}k'_{\perp}))P_{J+1}(z)+\right. \\ &+ \left[\sigma^2V_S(s,s',(k_{\perp}k'_{\perp}))+\frac{\sigma^2\Delta^2}{\sqrt{ss'}}V_V(s,s',(k_{\perp}k'_{\perp}))- \right. \\ &- \left.\sqrt{ss'}V_A(s,s',(k_{\perp}k'_{\perp}))-2\Delta^2V_T(s,s',(k_{\perp}k'_{\perp}))\right]\sqrt{k_{\perp}^2}\sqrt{k'_{\perp}^2}P_J(z)+ \\ &+ \left[s'-\Delta^2+4\frac{J}{2J+1}k'_{\perp}{}^2\right]k_{\perp}^2V_V(s,s',(k_{\perp}k'_{\perp}))P_{J-1}(z)\Big). \end{aligned}$$

And for the right side of (37):

$$\begin{aligned} Z_{\mu_1\dots\mu_J,\beta'}^{(J-1)}(k_{\perp})B_{\beta'\alpha'}(s,s',(k_{\perp}k'_{\perp}))X_{\mu_1\dots\mu_J,\alpha'}^{(J+1)}(k'_{\perp}) &= 16\alpha(J)k_{\perp}^2\left(\sqrt{k_{\perp}^2}\sqrt{k'_{\perp}^2}\right)^{J-1}\times(42) \\ &\times\left(\left[s'-\Delta^2+4\frac{J+1}{2J+1}k'_{\perp}{}^2\right]k_{\perp}^2V_V(s,s',(k_{\perp}k'_{\perp}))P_{J+1}(z)+\right. \\ &+ \left[\sigma^2V_S(s,s',(k_{\perp}k'_{\perp}))+\frac{\sigma^2\Delta^2}{\sqrt{ss'}}V_V(s,s',(k_{\perp}k'_{\perp}))- \right. \\ &- \left.\sqrt{ss'}V_A(s,s',(k_{\perp}k'_{\perp}))-2\Delta^2V_T(s,s',(k_{\perp}k'_{\perp}))\right]\sqrt{k_{\perp}^2}\sqrt{k'_{\perp}^2}P_J(z)+ \\ &+ \left[s-\Delta^2+4\frac{J}{2J+1}k_{\perp}^2\right]k'_{\perp}{}^2V_V(s,s',(k_{\perp}k'_{\perp}))P_{J-1}(z)\Big). \end{aligned}$$

and

$$\begin{aligned} Z_{\mu_1\dots\mu_J,\beta'}^{(J-1)}(k_{\perp})B_{\beta'\alpha'}(s,s',(k_{\perp}k'_{\perp}))Z_{\mu_1\dots\mu_J,\alpha'}^{(J-1)}(k'_{\perp}) &= 4\alpha(J)\left(\sqrt{k_{\perp}^2}\sqrt{k'_{\perp}^2}\right)^{J-1}\times(43) \\ &\times\left(16\frac{J+1}{2J+1}k_{\perp}^2k'_{\perp}{}^2V_V(s,s',(k_{\perp}k'_{\perp}))P_{J+1}(z)+\right. \\ &+ 4\left[\sigma^2V_S(s,s',(k_{\perp}k'_{\perp}))+\frac{\sigma^2\Delta^2}{\sqrt{ss'}}V_V(s,s',(k_{\perp}k'_{\perp}))+\right. \end{aligned}$$

$$\begin{aligned}
& + \frac{J+1}{J} \left(\sqrt{ss'} V_A(s, s', (k_\perp k'_\perp)) + 2\Delta^2 V_T(s, s', (k_\perp k'_\perp)) \right) \left[\sqrt{k_\perp^2} \sqrt{k'_\perp{}^2} P_J(z) + \right. \\
& + \left(\left[\frac{2J+1}{J} (s - \Delta^2)(s' - \Delta^2) \left(V_V(s, s', (k_\perp k'_\perp)) + 2 \frac{\sigma^2}{\sqrt{ss'}} V_T(s, s', (k_\perp k'_\perp)) \right) \right. \right. \\
& + 4(s' - \Delta^2) k_\perp^2 V_V(s, s', (k_\perp k'_\perp)) + 4(s - \Delta^2) k'_\perp{}^2 V_V(s, s', (k_\perp k'_\perp)) + \\
& \left. \left. + 16 \frac{J}{2J+1} k_\perp^2 k'_\perp{}^2 V_V(s, s', (k_\perp k'_\perp)) \right] P_{J-1}(z) \right].
\end{aligned}$$

In the right-hand sides of eqs. (36) and (37), we expand the interaction block in the Legendre polynomials series and integrate over angle variables $\int_{-1}^1 dz/2$. As a result, Eq. (36) reads:

$$\begin{aligned}
& (s - M^2) \left[4\psi_n^{(1, J-1, J)}(s) A_j + \left(\frac{2J+1}{J+1} (s - \Delta^2) + 4k_\perp^2 \right) \psi_n^{(1, J+1, J)}(s) B_j \right] = (44) \\
& = \int_{(m_1+m_2)^2}^{\infty} \frac{ds'}{\pi} \rho(s') 8 (-k'_\perp{}^2)^{J-1} \psi_n^{(1, J-1, J)}(s') A_j \times \\
& \times \left[\xi(J+1) k'_\perp{}^4 \left(s - \Delta^2 + 4 \frac{J+1}{2J+1} k_\perp^2 \right) \tilde{V}_V^{(J+1)}(s, s') - \right. \\
& - \xi(J) k'_\perp{}^2 \left[\sigma^2 \tilde{V}_S^{(J)}(s, s') + \frac{\sigma^2 \Delta^2}{\sqrt{ss'}} \tilde{V}_V^{(J)}(s, s') - \right. \\
& \left. - \sqrt{ss'} \tilde{V}_A^{(J)}(s, s') - 2\Delta^2 \tilde{V}_T^{(J)}(s, s') \right] + \\
& + \xi(J-1) \left[s' - \Delta^2 + 4 \frac{J}{2J+1} k'_\perp{}^2 \right] \tilde{V}_V^{(J-1)}(s, s') \Big) + \\
& + \int_{(m_1+m_2)^2}^{\infty} \frac{ds'}{\pi} \rho(s') 2 (-k'_\perp{}^2)^{J+1} \psi_n^{(1, J+1, J)}(k'_\perp{}^2) B_j \times \\
& \times \left(\xi(J+1) \left[\frac{2J+1}{J+1} (s - \Delta^2)(s' - \Delta^2) \left(\tilde{V}_V^{(J+1)}(s, s') + 2 \frac{\sigma^2}{\sqrt{ss'}} \tilde{V}_T^{(J+1)}(s, s') \right) \right. \right. \\
& + 4(s' - \Delta^2) k_\perp^2 \tilde{V}_V^{(J+1)}(s, s') + 4(s - \Delta^2) k'_\perp{}^2 \tilde{V}_V^{(J+1)}(s, s') + \\
& \left. \left. + 16 \frac{J+1}{2J+1} k_\perp^2 k'_\perp{}^2 \tilde{V}_V^{(J+1)}(s, s') \right] - \right.
\end{aligned}$$

$$\begin{aligned}
& -4\xi(J) \left[\sigma^2 \tilde{V}_S^{(J)}(s, s') + \frac{\sigma^2 \Delta^2}{\sqrt{ss'}} \tilde{V}_V^{(J)}(s, s') + \right. \\
& \left. + \frac{J}{J+1} \left(\sqrt{ss'} \tilde{V}_A^{(J)}(s, s') + 2\Delta^2 \tilde{V}_T^{(J)}(s, s') \right) \right] + \\
& + 16\xi(J-1) \frac{J}{2J+1} \tilde{V}_V^{(J-1)}(s, s').
\end{aligned}$$

The second equation, (37), reads:

$$\begin{aligned}
& (s - M^2) \left[\left(\frac{2J+1}{J} (s - \Delta^2) + 4k_\perp^2 \right) \psi_n^{(1, J-1, J)}(s) A_j + 4k_\perp^4 \psi_n^{(1, J+1, J)}(s) B_j \right] = (45) \\
& = \int_{(m_1+m_2)^2}^{\infty} \frac{ds'}{\pi} \rho(s') 2 (-k_\perp'^2)^{J-1} \psi_n^{(1, J-1, J)}(s') A_j \times \\
& \quad \times \left(16\xi(J+1) \frac{J+1}{2J+1} k_\perp^4 k_\perp'^4 \tilde{V}_V^{(J+1)}(s, s') - \right. \\
& \quad - 4\xi(J) k_\perp^2 k_\perp'^2 \left[\sigma^2 \tilde{V}_S^{(J)}(s, s') + \frac{\sigma^2 \Delta^2}{\sqrt{ss'}} \tilde{V}_V^{(J)}(s, s') + \right. \\
& \quad \left. \left. + \frac{J+1}{J} \left(\sqrt{ss'} \tilde{V}_A^{(J)}(s, s') + 2\Delta^2 \tilde{V}_T^{(J)}(s, s') \right) \right] - \right. \\
& \quad + \xi(J-1) \left(\left[\frac{2J+1}{J} (s - \Delta^2) (s' - \Delta^2) \left(\tilde{V}_V^{(J-1)}(s, s') + 2 \frac{\sigma^2}{\sqrt{ss'}} \tilde{V}_T^{(J-1)}(s, s') \right) \right] + \right. \\
& \quad + 4(s' - \Delta^2) k_\perp^2 \tilde{V}_V^{(J-1)}(s, s') + 4(s - \Delta^2) k_\perp'^2 \tilde{V}_V^{(J-1)}(s, s') + \\
& \quad \left. + 16 \frac{J}{2J+1} k_\perp^2 k_\perp'^2 \tilde{V}_V^{(J-1)}(s, s') \right] + \\
& \quad + \int_{(m_1+m_2)^2}^{\infty} \frac{ds'}{\pi} \rho(s') 8 (-k_\perp'^2)^{J+1} \psi_n^{(1, J+1, J)}(s') B_j \times \\
& \quad \times \xi(J+1) k_\perp^4 \left(\left[s' - \Delta^2 + 4 \frac{J+1}{2J+1} k_\perp'^2 \right] \tilde{V}_V^{(J+1)}(s, s') - \right. \\
& \quad \left. - \xi(J) k_\perp^2 \left[\sigma^2 \tilde{V}_S^{(J)}(s, s') + \frac{\sigma^2 \Delta^2}{\sqrt{ss'}} \tilde{V}_V^{(J)}(s, s') - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& -\sqrt{ss'} \tilde{V}_A^{(J)}(s, s') - 2\Delta^2 \tilde{V}_T^{(J)}(s, s')] + \\
& + \xi(J-1) \left[s - \Delta^2 + 4\frac{J}{2J+1}k_\perp^2 \right] \tilde{V}_V^{(J-1)}(s, s') \Big).
\end{aligned}$$

Normalization and orthogonality conditions are :

$$\begin{aligned}
& \int_{(m_2+m_1)^2}^{\infty} \frac{ds}{\pi} \rho(s) \left[A_j^2 \left(\psi_n^{(1, J-1, J)}(k_\perp^2) \right)^2 2\alpha(J)(-k_\perp^2)^{(J-1)} \times \right. \\
& \quad \times \left(\frac{2J+1}{J}(s - \Delta^2) + 4k_\perp^2 \right) + \\
& \quad + 2A_j B_j \psi_n^{(1, J-1, J)}(k_\perp^2) \psi_n^{(1, J+1, J)}(k_\perp^2) 8\alpha(J)(-k_\perp^2)^{(J+1)} + \\
& \quad \left. + B_j^2 \left(\psi_n^{(1, J+1, J)}(k_\perp^2) \right)^2 2\alpha(J)(-k_\perp^2)^{(J+1)} \left(\frac{2J+1}{J+1}(s - \Delta^2) + 4k_\perp^2 \right) \right] = 1 \quad j = 1, 2,
\end{aligned} \tag{46}$$

and

$$\begin{aligned}
& \int_{(m_2-m_1)^2}^{\infty} \frac{ds}{\pi} \rho(s) \left[A_1 A_2 \left(\psi_n^{(1, J-1, J)}(k_\perp^2) \right)^2 2\alpha(J)(-k_\perp^2)^{(J-1)} \times \right. \\
& \quad \times \left(\frac{2J+1}{J}(s - \Delta^2) + 4k_\perp^2 \right) + \\
& \quad + (A_1 B_2 + A_2 B_1) \psi_n^{(1, J-1, J)}(k_\perp^2) \psi_n^{(1, J+1, J)}(k_\perp^2) 8\alpha(J)(-k_\perp^2)^{(J+1)} + \\
& \quad \left. + B_1 B_2 \left(\psi_n^{(1, J+1, J)}(k_\perp^2) \right)^2 2\alpha(J)(-k_\perp^2)^{(J+1)} \left(\frac{2J+1}{J+1}(s - \Delta^2) + 4k_\perp^2 \right) \right] = 0.
\end{aligned} \tag{47}$$

Let us emphasize again: all the above equations are written for $J > 0$.

4 Conclusion

We have presented the Bethe-Salpeter equations for the quark-antiquark systems when the quark and antiquark have different masses. The main difference from the equal mass case is that there is the mixture of states ($J = L, S = 0$) and ($J = L, S = 1$) and that is proportional to the quark mass difference. The mixing between ($S = 1, J = L$) and ($S = 0,$

$J = L$) states give rise to a strongly correlated system of equations. In the equation for states with total spin J we need to know all lower projections of the potential on the Legendre polynomials, not only the $J + 1, J, J - 1$ ones. The numerical study of this equations is now in progress.

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5 Appendix A: Traces for loop diagrams

Here we present the traces used in the calculation of loop diagrams. Recall that in the spectral integral representation, there is no energy conservation, $s \neq s'$, where $P^2 = s$, $P'^2 = s'$, but all constituents are mass-on-shell:

$$k_1^2 = m_1^2, \quad k_2^2 = m_2^2, \quad k_1'^2 = m_1^2, \quad k_2'^2 = m_2^2.$$

We have used notations for the quark momenta:

$$\begin{aligned} k_\nu &= \frac{1}{2}(k_1 - k_2)_\nu, \quad k'_\nu = \frac{1}{2}(k'_1 - k'_2)_\nu, \\ k_\mu^\perp &= k_\nu g_{\nu\mu}^\perp, \quad k'_\mu^\perp = k'_\nu g_{\nu\mu}^\perp, \\ k_\mu &= \frac{m_1^2 - m_2^2}{2s} P_\mu + k_\mu^\perp, \quad k'_\mu = \frac{m_1^2 - m_2^2}{2s'} P'_\mu + k'_\mu^\perp, \end{aligned} \tag{48}$$

and for the quarks masses :

$$\Delta = m_2 - m_1, \quad \sigma = m_2 + m_1, \quad . \tag{49}$$

We work with the following definition of the matrices:

$$\gamma_5 = -i\gamma^0\gamma^1\gamma^2\gamma^3, \quad \sigma_{\mu\nu} = \frac{1}{2}[\gamma_\mu\gamma_\nu].$$

5.1 Traces for the $S = 0$ states

For the $S = 0$ states we have the following non-zero traces:

$$T'_P = Tr \left[i\gamma_5(\hat{k}'_1 + m_1)\gamma_5(-\hat{k}'_2 + m_2) \right] = 2i(s' - (m_2 - m_1)^2), \tag{50}$$

$$\begin{aligned}
T'_A &= Tr \left[i\gamma_5(\hat{k}'_1 + m_1)i\gamma_\mu\gamma_5(-\hat{k}'_2 + m_2) \right] = -2 \left[2k'_\mu(m_2 - m_1) + P'_\mu(m_2 + m_1) \right], \\
T'_T &= Tr \left[i\gamma_5(\hat{k}'_1 + m_1)i\sigma_{\mu\nu}(-\hat{k}'_2 + m_2) \right] = -4i\epsilon_{\mu\nu\alpha\beta}P'_\alpha k'_\beta,
\end{aligned}$$

and

$$\begin{aligned}
T_P &= Tr \left[i\gamma_5(-\hat{k}_2 + m_2)\gamma_5(\hat{k}_1 + m_1) \right] = 2i(s - (m_2 - m_1)^2), \\
T_A &= Tr \left[i\gamma_5(-\hat{k}_2 + m_2)i\gamma_\mu\gamma_5(\hat{k}_1 + m_1) \right] = 2 \left[2k_\mu(m_2 - m_1) + P_\mu(m_2 + m_1) \right], \\
T_T &= Tr \left[i\gamma_5(-\hat{k}_2 + m_2)i\sigma_{\mu\nu}(\hat{k}_1 + m_1) \right] = 4i\epsilon_{\mu\nu\alpha\beta}P_\alpha k_\beta.
\end{aligned} \tag{51}$$

The convolutions of the traces $A_P = (T_P T'_P)$, $A_A = (T_A T'_A)$, $A_T = (T_T T'_T)$ are equal to:

$$\begin{aligned}
A_P &= -4(s - \Delta^2)(s' - \Delta^2), \\
A_A &= -16\Delta^2 \sqrt{k_\perp^2 k_\perp'^2} z - 4 \frac{\sigma^2}{\sqrt{ss'}} (s - \Delta^2)(s' - \Delta^2), \\
A_T &= -32\sqrt{ss'} \sqrt{k_\perp^2 k_\perp'^2} z.
\end{aligned} \tag{52}$$

5.2 Traces for the $S = 1$ states

For the $(S = 1)$ -states, the traces are equal to:

$$\begin{aligned}
T'_S &= Tr \left[\gamma_{\alpha'}^\perp(\hat{k}'_1 + m_1)(-\hat{k}'_2 + m_2) \right] = 4k_{\alpha'}^\perp(m_2 + m_1), \\
T'_V &= Tr \left[\gamma_{\alpha'}^\perp(\hat{k}'_1 + m_1)\gamma_\mu(-\hat{k}'_2 + m_2) \right] = 2 \left(g_{\alpha'\mu}^\perp(s' - (m_2 - m_1)^2) + 4k_{\alpha'}^\perp k'_\mu \right), \\
T'_A &= Tr \left[\gamma_{\alpha'}^\perp(\hat{k}'_1 + m_1)i\gamma_\mu^\perp\gamma_5(-\hat{k}'_2 + m_2) \right] = 4\epsilon_{\alpha'\mu\alpha\beta}k'_\alpha P'_\beta, \\
T'_T &= Tr \left[\gamma_{\alpha'}^\perp(\hat{k}'_1 + m_1)i\sigma_{\mu\nu}(-\hat{k}'_2 + m_2) \right] = \\
&= 2i \left[2(m_2 - m_1) \left(g_{\alpha'\nu}^\perp k'_\mu - g_{\alpha'\mu}^\perp k'_\nu \right) + (m_1 + m_2) \left(g_{\alpha'\nu}^\perp P'_\mu - g_{\alpha'\mu}^\perp P'_\nu \right) \right],
\end{aligned} \tag{53}$$

and

$$\begin{aligned}
T_S &= Tr \left[\gamma_{\beta'}^\perp(-\hat{k}_2 + m_2)\hat{k}_1 + m_1 \right] = 4k_{\beta'}^\perp(m_1 + m_2), \\
T_V &= Tr \left[\gamma_{\beta'}^\perp(-\hat{k}_2 + m_2)\gamma_\mu(\hat{k}_1 + m_1) \right] = 2 \left[g_{\mu\beta'}^\perp(s - (m_2 - m_1)^2) + 4k_{\beta'}^\perp k_\mu \right], \\
T_A &= Tr \left[\gamma_{\beta'}^\perp(-\hat{k}_2 + m_2)i\gamma_\mu\gamma_5(\hat{k}_1 + m_1) \right] = -4\epsilon_{\beta'\mu\alpha\beta}k_\alpha P_\beta,
\end{aligned} \tag{54}$$

$$\begin{aligned}
T_T &= Tr \left[\gamma_{\beta'}^\perp (-\hat{k}_2 + m_2) i \sigma_{\mu\nu} (\hat{k}_1 + m_1) \right] = \\
&= 2i \left[2(m_2 - m_1) (g_{\beta'\mu}^\perp k_\nu - g_{\beta'\nu}^\perp k_\mu) + (m_2 + m_1) (g_{\beta'\mu}^\perp P_\nu - g_{\beta'\nu}^\perp P_\mu) \right].
\end{aligned}$$

Corresponding convolution $B_S = (T_c T'_c)$ reads :

$$\begin{aligned}
(B_S)_{\beta'\alpha'} &= 16 k_{\beta'}^\perp k_{\alpha'}'^\perp \sigma^2, \\
(B_V)_{\beta'\alpha'} &= 4 \left[(g_{\beta'\alpha'}^\perp (s - \Delta^2) (s' - \Delta^2) + 16 k_{\beta'}^\perp k_{\alpha'}'^\perp \sqrt{k_\perp^2 k_\perp'^2} z + \right. \\
&\quad \left. + 4 k_{\beta'}^\perp k_{\alpha'}^\perp (s' - \Delta^2) + 4 k_{\beta'}'^\perp k_{\alpha'}'^\perp (s - \Delta^2) + 4 k_{\beta'}^\perp k_{\alpha'}'^\perp \frac{\sigma^2 \Delta^2}{\sqrt{ss'}} \right], \\
(B_A)_{\beta'\alpha'} &= -16 \sqrt{ss'} \left[k_{\beta'}'^\perp k_{\alpha'}^\perp - g_{\beta'\alpha'}^\perp \sqrt{k_\perp^2 k_\perp'^2} z \right], \\
(B_T)_{\beta'\alpha'} &= -8 \left[4 \Delta^2 \left(k_{\beta'}'^\perp k_{\alpha'}^\perp - g_{\beta'\alpha'}^\perp \sqrt{k_\perp^2 k_\perp'^2} z \right) - g_{\beta'\alpha'}^\perp \frac{\sigma^2}{\sqrt{ss'}} (s - \Delta^2) (s' - \Delta^2) \right].
\end{aligned} \tag{55}$$

6 Appendix B: Convolutions of trace factors

Here we present the convolutions of the angular-momentum factors. Let z be:

$$z = \frac{(k_\perp k_\perp')}{\sqrt{k_\perp^2} \sqrt{k_\perp'^2}}. \tag{56}$$

The convolutions for the ($S = 1$) states read:

$$X_{\mu_1 \mu_2 \dots \mu_J}^{(J)}(k_\perp) X_{\mu_1 \mu_2 \dots \mu_J}^{(J)}(k'_\perp) = \alpha(J) \left(\sqrt{k_\perp^2} \sqrt{k_\perp'^2} \right)^J P_J(z). \tag{57}$$

Analogous convolutions for ($S = 1$)-states are written as follows:

$$\begin{aligned}
X_{\mu_1 \mu_2 \dots \mu_J \beta}^{(J+1)}(k_\perp) X_{\mu_1 \mu_2 \dots \mu_J \alpha}^{(J+1)}(k'_\perp) &= \frac{\alpha(J)}{J+1} \left(\sqrt{k_\perp^2} \sqrt{k_\perp'^2} \right)^J \times \\
&\times \left[\frac{\sqrt{k_\perp'^2}}{\sqrt{k_\perp^2}} A_{P_{J,J+1}}(z) k_\beta^\perp k_\alpha^\perp + \frac{\sqrt{k_\perp^2}}{\sqrt{k_\perp'^2}} B_{P_{J,J+1}}(z) k_\beta'^\perp k_\alpha'^\perp + \right. \\
&\left. + C_{P_{J,J+1}}(z) k_\beta^\perp k_\alpha'^\perp + D_{P_{J,J+1}}(z) k_\beta'^\perp k_\alpha^\perp + \left(\sqrt{k_\perp^2} \sqrt{k_\perp'^2} \right) E_{P_{J,J+1}}(z) g_{\beta\alpha}^\perp \right],
\end{aligned} \tag{58}$$

$$\begin{aligned}
X_{\mu_1\mu_2\cdots\mu_J\beta}^{(J+1)}(k_\perp) Z_{\mu_1\mu_2\cdots\mu_J,\alpha}^{(J-1)}(k'_\perp) &= -\frac{\alpha(J)}{J} \frac{1}{k_\perp'^2} \left(\sqrt{k_\perp^2} \sqrt{k_\perp'^2} \right)^J \times \\
&\times \left[\frac{\sqrt{k_\perp'^2}}{\sqrt{k_\perp^2}} A_{P_{J,J+1}}(z) k_\beta^\perp k_\alpha^\perp + \frac{\sqrt{k_\perp^2}}{\sqrt{k_\perp'^2}} \left(B_{P_{J,J+1}}(z) - (2J+1)A_J(z) \right) k_\beta'^\perp k_\alpha'^\perp + \right. \\
&+ (C_{P_{J,J+1}}(z) - (2J+1)B_J(z)) k_\beta^\perp k_\alpha'^\perp + \\
&\left. + D_{P_{J,J+1}}(z) k_\beta'^\perp k_\alpha^\perp + \left(\sqrt{k_\perp^2} \sqrt{k_\perp'^2} \right) E_{P_{J,J+1}}(z) g_{\beta\alpha}^\perp \right], \quad (59)
\end{aligned}$$

$$\begin{aligned}
Z_{\mu_1\mu_2\cdots\mu_J,\beta}^{(J-1)}(k_\perp) Z_{\mu_1\mu_2\cdots\mu_J,\alpha}^{(J-1)}(k'_\perp) &= \frac{J+1}{J^2} \alpha(J) \left(\sqrt{k_\perp^2} \sqrt{k_\perp'^2} \right)^{J-2} \times \\
&\times \left[\frac{\sqrt{k_\perp'^2}}{\sqrt{k_\perp^2}} \left(A_{P_{J,J+1}}(z) - (2J+1)A_J(z) \right) k_\beta^\perp k_\alpha^\perp + \right. \\
&+ \frac{\sqrt{k_\perp^2}}{\sqrt{k_\perp'^2}} \left(B_{P_{J,J+1}}(z) - (2J+1)A_J(z) \right) k_\beta'^\perp k_\alpha'^\perp + \\
&+ \left(C_{P_{J,J+1}}(z) + \frac{(2J+1)^2}{J+1} P_J(z) - 2(2J+1)B_J(z) \right) k_\beta^\perp k_\alpha'^\perp + \\
&\left. + D_{P_{J,J+1}}(z) k_\beta'^\perp k_\alpha^\perp + \left(\sqrt{k_\perp^2} \sqrt{k_\perp'^2} \right) E_{P_{J,J+1}}(z) g_{\beta\alpha}^\perp \right], \quad (60)
\end{aligned}$$

$$\begin{aligned}
&\epsilon_{\beta\nu_1\nu_2\nu_3} P_{\nu_1} Z_{\nu_2\mu_1\cdots\mu_J,\nu_3}^{(J)}(k_\perp) \epsilon_{\alpha\lambda_1\lambda_2\lambda_3} P'_{\lambda_1} Z_{\lambda_2\mu_1\cdots\mu_J,\lambda_3}^{(J)}(k'_\perp) \quad (61) \\
&= \frac{(2J+3)^2}{(J+1)^3} \alpha(J) \left(\sqrt{k_\perp^2} \sqrt{k_\perp'^2} \right)^{J-1} (PP') \times \\
&\times \left[-\sqrt{k_\perp^2} \sqrt{k_\perp'^2} \left((z^2-1)D_{P_{J,J+1}}(z) + zE_{P_{J,J+1}}(z) \right) g_{\beta\alpha}^\perp - \right. \\
&- D_{P_{J,J+1}}(z) \left(\frac{\sqrt{k_\perp'^2}}{\sqrt{k_\perp^2}} k_\beta^\perp k_\alpha^\perp + \frac{\sqrt{k_\perp^2}}{\sqrt{k_\perp'^2}} k_\beta'^\perp k_\alpha'^\perp - z k_\beta^\perp k_\alpha'^\perp \right) + \\
&\left. + \left(zD_{P_{J,J+1}}(z) + E_{P_{J,J+1}}(z) \right) k_\beta'^\perp k_\alpha^\perp \right],
\end{aligned}$$

and

$$X_{\mu_1\cdots\mu_J}^{(J)}(k_\perp) \epsilon_{\alpha\nu_1\nu_2\nu_3} P'_{\nu_1} Z_{\nu_2\mu_1\cdots\mu_J,\nu_3}^{(J)}(k'_\perp) = \quad (62)$$

$$= \frac{2J+3}{J+1} \alpha(J) \left(\sqrt{k_{\perp}^2} \sqrt{k'_{\perp}{}^2} \right)^{J-1} A_{P_J, J+1}(z) \epsilon_{\alpha P' k k'}.$$

Here

$$\begin{aligned} A_{P_J, J+1}(z) &= B_{P_J, J+1}(z) = -\frac{2zP_J(z) + [Jz^2 - (J+2)]P_{J+1}(z)}{(1-z^2)^2}, \\ C_{P_J, J+1}(z) &= \frac{[(1-J)z^2 + (J+1)]P_J(z) + [(2J+1)z^2 - (2J+3)]zP_{J+1}(z)}{(1-z^2)^2}, \\ D_{P_J, J+1}(z) &= \frac{[(J+2)z^2 - J]P_J(z) - 2zP_{J+1}(z)}{(1-z^2)^2}, \\ E_{P_J, J+1}(z) &= \frac{zP_J(z) - P_{J+1}(z)}{(1-z^2)}. \end{aligned} \quad (63)$$

For the factors

$K_{\beta} X_{\mu_1 \mu_2 \dots \mu_J \beta}^{(J+1)}(k_{\perp}) X_{\mu_1 \mu_2 \dots \mu_J \alpha}^{(J+1)}(k'_{\perp}) K_{\alpha}$, where $K = k, k'$, we need more complicated convolutions, namely: :

$$\begin{aligned} k_{\beta} X_{\mu_1 \mu_2 \dots \mu_J \beta}^{(J+1)}(k_{\perp}) X_{\mu_1 \mu_2 \dots \mu_J \alpha}^{(J+1)}(k'_{\perp}) k_{\alpha} &= k_{\perp}^2 \alpha(J) \left(\sqrt{k_{\perp}^2} \sqrt{k'_{\perp}{}^2} \right)^{J+1} P_{J+1}(z), \\ k_{\beta} X_{\mu_1 \mu_2 \dots \mu_J \beta}^{(J+1)}(k_{\perp}) X_{\mu_1 \mu_2 \dots \mu_J \alpha}^{(J+1)}(k'_{\perp}) k'_{\alpha} &= \alpha(J) \left(\sqrt{k_{\perp}^2} \sqrt{k'_{\perp}{}^2} \right)^{J+2} P_J(z), \\ k'_{\beta} X_{\mu_1 \mu_2 \dots \mu_J \beta}^{(J+1)}(k_{\perp}) X_{\mu_1 \mu_2 \dots \mu_J \alpha}^{(J+1)}(k'_{\perp}) k'_{\alpha} &= k'_{\perp}^2 \alpha(J) \left(\sqrt{k_{\perp}^2} \sqrt{k'_{\perp}{}^2} \right)^{J+1} P_{J+1}(z), \\ k'_{\beta} X_{\mu_1 \mu_2 \dots \mu_J \beta}^{(J+1)}(k_{\perp}) X_{\mu_1 \mu_2 \dots \mu_J \alpha}^{(J+1)}(k'_{\perp}) k_{\alpha} &= \\ &= \alpha(J) \left(\sqrt{k_{\perp}^2} \sqrt{k'_{\perp}{}^2} \right)^{J+2} \left[\frac{2J+1}{J+1} z P_{J+1}(z) - \frac{J}{J+1} P_J(z) \right], \\ g_{\beta \alpha}^{\perp} X_{\mu_1 \mu_2 \dots \mu_J \beta}^{(J+1)}(k_{\perp}) X_{\mu_1 \mu_2 \dots \mu_J \alpha}^{(J+1)}(k'_{\perp}) &= \frac{2J+1}{J+1} \alpha(J) \left(\sqrt{k_{\perp}^2} \sqrt{k'_{\perp}{}^2} \right)^{J+1} P_{J+1}(z), \end{aligned}$$

as well as for the factors $K_{\beta} X_{\mu_1 \mu_2 \dots \mu_J \beta}^{(J+1)}(k_{\perp}) Z_{\mu_1 \mu_2 \dots \mu_J, \alpha}^{(J-1)}(k'_{\perp}) K_{\alpha}$:

$$\begin{aligned} k_{\beta} X_{\mu_1 \mu_2 \dots \mu_J \beta}^{(J+1)}(k_{\perp}) Z_{\mu_1 \mu_2 \dots \mu_J, \alpha}^{(J-1)}(k'_{\perp}) k_{\alpha} &= k_{\perp}^4 \alpha(J) \left(\sqrt{k_{\perp}^2} \sqrt{k'_{\perp}{}^2} \right)^{J-1} P_{J-1}(z), \\ k_{\beta} X_{\mu_1 \mu_2 \dots \mu_J \beta}^{(J+1)}(k_{\perp}) Z_{\mu_1 \mu_2 \dots \mu_J, \alpha}^{(J-1)}(k'_{\perp}) k'_{\alpha} &= k_{\perp}^2 \alpha(J) \left(\sqrt{k_{\perp}^2} \sqrt{k'_{\perp}{}^2} \right)^J P_J(z), \end{aligned}$$

$$\begin{aligned}
k'_\beta X_{\mu_1\mu_2\cdots\mu_J\beta}^{(J+1)}(k_\perp) Z_{\mu_1\mu_2\cdots\mu_J,\alpha}^{(J-1)}(k'_\perp) k'_\alpha &= \alpha(J) \left(\sqrt{k_\perp^2} \sqrt{k'^2_\perp} \right)^{J+1} P_{J+1}(z), \\
k'_\beta X_{\mu_1\mu_2\cdots\mu_J\beta}^{(J+1)}(k_\perp) Z_{\mu_1\mu_2\cdots\mu_J,\alpha}^{(J-1)}(k'_\perp) k_\alpha &= k_\perp^2 \alpha(J) \left(\sqrt{k_\perp^2} \sqrt{k'^2_\perp} \right)^J P_J(z), \\
g^\perp_{\beta\alpha} X_{\mu_1\mu_2\cdots\mu_J\beta}^{(J+1)}(k_\perp) Z_{\mu_1\mu_2\cdots\mu_J,\alpha}^{(J-1)}(k'_\perp) &= 0,
\end{aligned}$$

and for the factors $K_\beta Z_{\mu_1\mu_2\cdots\mu_J,\beta}^{(J-1)}(k_\perp) Z_{\mu_1\mu_2\cdots\mu_J,\alpha}^{(J-1)}(k'_\perp) K_\alpha$:

$$\begin{aligned}
k_\beta Z_{\mu_1\mu_2\cdots\mu_J,\beta}^{(J-1)}(k_\perp) Z_{\mu_1\mu_2\cdots\mu_J,\alpha}^{(J-1)}(k'_\perp) k_\alpha &= k_\perp^2 \alpha(J) \left(\sqrt{k_\perp^2} \sqrt{k'^2_\perp} \right)^{J-1} P_{J-1}(z), \\
k_\beta Z_{\mu_1\mu_2\cdots\mu_J,\beta}^{(J-1)}(k_\perp) Z_{\mu_1\mu_2\cdots\mu_J,\alpha}^{(J-1)}(k'_\perp) k'_\alpha &= \alpha(J) \left(\sqrt{k_\perp^2} \sqrt{k'^2_\perp} \right)^J P_J(z), \\
k'_\beta Z_{\mu_1\mu_2\cdots\mu_J,\beta}^{(J-1)}(k_\perp) Z_{\mu_1\mu_2\cdots\mu_J,\alpha}^{(J-1)}(k'_\perp) k'_\alpha &= k'^2_\perp \alpha(J) \left(\sqrt{k_\perp^2} \sqrt{k'^2_\perp} \right)^{J-1} P_{J-1}(z), \\
k'_\beta Z_{\mu_1\mu_2\cdots\mu_J,\beta}^{(J-1)}(k_\perp) Z_{\mu_1\mu_2\cdots\mu_J,\alpha}^{(J-1)}(k'_\perp) k_\alpha \\
&= \alpha(J) \left(\sqrt{k_\perp^2} \sqrt{k'^2_\perp} \right)^J \left[\frac{2J+1}{J} z P_{J-1}(z) - \frac{J+1}{J} P_J(z) \right], \\
g^\perp_{\beta\alpha} Z_{\mu_1\mu_2\cdots\mu_J,\beta}^{(J-1)}(k_\perp) Z_{\mu_1\mu_2\cdots\mu_J,\alpha}^{(J-1)}(k'_\perp) &= \frac{2J+1}{J} \alpha(J) \left(\sqrt{k_\perp^2} \sqrt{k'^2_\perp} \right)^{J-1} P_{J-1}(z),
\end{aligned}$$

and for the factors $K_\beta \epsilon_{\beta\nu_1\nu_2\nu_3} P_{\nu_1} Z_{\nu_2\mu_1\cdots\mu_J,\nu_3}^{(J)}(k_\perp) \epsilon_{\alpha\lambda_1\lambda_2\lambda_3} P'_{\lambda_1} Z_{\lambda_2\mu_1\cdots\mu_J,\lambda_3}^{(J)}(k'_\perp) K_\alpha$:

$$\begin{aligned}
k_\beta \epsilon_{\beta\nu_1\nu_2\nu_3} P_{\nu_1} Z_{\nu_2\mu_1\cdots\mu_J,\nu_3}^{(J)}(k_\perp) \epsilon_{\alpha\lambda_1\lambda_2\lambda_3} P'_{\lambda_1} Z_{\lambda_2\mu_1\cdots\mu_J,\lambda_3}^{(J)}(k'_\perp) k_\alpha &= 0, \\
k_\beta \epsilon_{\beta\nu_1\nu_2\nu_3} P_{\nu_1} Z_{\nu_2\mu_1\cdots\mu_J,\nu_3}^{(J)}(k_\perp) \epsilon_{\alpha\lambda_1\lambda_2\lambda_3} P'_{\lambda_1} Z_{\lambda_2\mu_1\cdots\mu_J,\lambda_3}^{(J)}(k'_\perp) k'_\alpha &= 0, \\
k'_\beta \epsilon_{\beta\nu_1\nu_2\nu_3} P_{\nu_1} Z_{\nu_2\mu_1\cdots\mu_J,\nu_3}^{(J)}(k_\perp) \epsilon_{\alpha\lambda_1\lambda_2\lambda_3} P'_{\lambda_1} Z_{\lambda_2\mu_1\cdots\mu_J,\lambda_3}^{(J)}(k'_\perp) k'_\alpha &= 0, \\
k'_\beta \epsilon_{\beta\nu_1\nu_2\nu_3} P_{\nu_1} Z_{\nu_2\mu_1\cdots\mu_J,\nu_3}^{(J)}(k_\perp) \epsilon_{\alpha\lambda_1\lambda_2\lambda_3} P'_{\lambda_1} Z_{\lambda_2\mu_1\cdots\mu_J,\lambda_3}^{(J)}(k'_\perp) k_\alpha \\
&= \frac{(2J+3)^2}{(J+1)^3} \alpha(J) \left(\sqrt{k_\perp^2} \sqrt{k'^2_\perp} \right)^{J+1} (PP') [z P_J(z) - P_{J+1}(z)], \\
g^\perp_{\beta\alpha} \epsilon_{\beta\nu_1\nu_2\nu_3} P_{\nu_1} Z_{\nu_2\mu_1\cdots\mu_J,\nu_3}^{(J)}(k_\perp) \epsilon_{\alpha\lambda_1\lambda_2\lambda_3} P'_{\lambda_1} Z_{\lambda_2\mu_1\cdots\mu_J,\lambda_3}^{(J)}(k'_\perp) &= \\
&= -\frac{J(2J+3)^2}{(J+1)^3} \alpha(J) \left(\sqrt{k_\perp^2} \sqrt{k'^2_\perp} \right)^J (PP') P_J(z).
\end{aligned}$$

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